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Original Research

Conceptual Gap Analysis in Learning Fractions: A Diagnostic Study of Elementary School Students' Cognitive Structure

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Abstract: This study aims to analyze and map students' cognitive structures in fractional number operations that cause conceptual gaps and recommend effective learning intervention strategies to address these gaps. This study used a descriptive and correlational research design involving 23 sixth-grade students (12 boys, 11 girls) selected using a purposive sampling technique. The developed test instrument was used as an instrument to collect data on students' conceptual gaps in fractional number operations. The findings indicate that students show less conceptual understanding in fractional number operations. The higher the level of students' conceptual understanding, the higher the mathematical competence they demonstrate. The pedagogical implications of this study emphasize the need for cognitive mapping-based interventions for elementary school teachers to mitigate students' epistemological barriers in understanding the abstract concept of fractions.

Abstrak: Penelitian ini bertujuan untuk menganalisis dan memetakan struktur kognitif siswa dalam operasi bilangan pecahan yang menyebabkan kesenjangan konseptual, dan merekomendasikan strategi intervensi pembelajaran yang efektif guna mengatasi kesenjangan tersebut. Penelitian ini menggunakan desain penelitian deskriptif dan korelasional yang melibatkan 23 siswa kelas 6 (laki-laki 12 orang, perempuan 11 orang) yang dipilih menggunakan teknik *purposive sampling*. Instrumen tes yang telah dikembangkan digunakan sebagai instrumen untuk mengambil data kesenjangan konseptual siswa pada operasi bilangan pecahan. Temuan menunjukkan bahwa siswa memiliki pemahaman konseptual yang rendah dalam operasi bilangan pecahan. Selain itu, ditemukan bahwa semakin tinggi tingkat pemahaman konseptual siswa, semakin tinggi pula kompetensi matematika yang mereka tunjukkan. Implikasi pedagogis dari penelitian ini menekankan perlunya intervensi berbasis pemetaan kognitif bagi guru sekolah dasar untuk memitigasi hambatan epistemologis siswa dalam memahami konsep abstrak bilangan pecahan.

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INTRODUCTION

Conceptual understanding in mathematics is the main foundation in developing thinking and problem-solving skills at the elementary and secondary school levels (Al-Mutawah et al., 2019; ATINUKE et al., 2024). This is inseparable from the positive relationship between conceptual knowledge and academic achievement in mathematics (Andamon & Tan, 2018; Hatisaru & Erbas, 2017; Kanwal et al., 2022). This relationship shows that the better students' conceptual knowledge of the basic structures and principles of mathematics, the better their problem-solving abilities, which ultimately lead to optimal academic scores. Students with a good conceptual understanding are able to connect comprehensive and functional mathematical ideas that are useful in problem-solving (Eli et al., 2013; Kholid et al., 2021). However, one topic that has consistently proven to be a major problem for elementary and secondary school students is the concept of fractions (fraction concepts). Helping in mastering the concept of fractions not only hinders students' academic progress, but is also often identified as a strong predictor of the emergence of the phenomenon of mathematics anxiety in students (Ansari & Wahyu, 2017; Khoule et al., 2017).

Literature studies indicate a variety of conceptual errors found in students' understanding of fractions. These errors include the inability to relate the concept of part to whole (Aksoy & Yazlik, 2017; Sinnakaudan & Ghazali, 2015), errors in visualizing or modeling a fraction (Anderson-Pence et al., 2014; Ergul & Tertemiz, 2024), and serious misconceptions in implementing fraction arithmetic operations (Kukulaje et al., 2024; Ratnasari, 2018; Üzel, 2018). Therefore, learning requires a shift in focus. An approach that prioritizes conceptual understanding over algorithmic procedures is needed, for example by utilizing concrete models and connecting fraction concepts to real-life contexts to create a more meaningful and sustainable learning process.

Several studies have shown that developing students' conceptual understanding can influence their comprehension (Cummings, 2015; Diyana & Sutopo, 2024), and improving students' conceptual understanding can also improve their procedural fluency (Cummings, 2015; Polly & Martin, 2025). However, a more assertive view holds that solid conceptual mastery inherently guarantees fluent problem-solving (Yuan, 2013). These nuanced differences indicate a theoretical gap regarding the mechanisms and nature of the causal relationship between conceptual understanding and problem-solving, particularly in the context of specific materials such as fraction operations. This theoretical gap creates an empirical gap regarding how to precisely identify the root causes of learning failures related to conceptual understanding, which can then be addressed through specifically designed interventions. A deeper understanding of this conceptual-procedural relationship is crucial for formulating more effective teaching strategies that can equip students with the foundational skills needed to tackle more complex mathematical tasks. Addressing this gap requires a robust diagnostic framework to scientifically identify the root causes of learning difficulties, not just their symptoms. The novelty of this research lies in its diagnostic approach, which integrates cognitive structure analysis with specific pedagogical intervention recommendations, an area still limited in the current literature.

This study specifically aims to analyze and map students' cognitive structures in fraction operations that cause conceptual gaps. Furthermore, it recommends effective learning intervention strategies to address these gaps. This research provides practical contributions that can be immediately implemented. The results of this study are expected to make a substantial contribution to the mathematics education literature, by providing empirical insights for educators and policymakers, transforming failures in abstract understanding into solid conceptual mastery, and providing a strong foundation for the development of more focused curricula and pedagogies.

Conceptual framework

Figure 1 shows the conceptual framework of this study. This research is fundamentally based on mapping and analyzing the cognitive structures of sixth-grade students related to fraction operations, with a primary focus on identifying conceptual gaps. The underlying assumption is that gaps in understanding and operations are directly related to cognitive structures that are not yet optimally organized. The method used combines a descriptive approach to document in detail the nature of these cognitive structures and gaps, and a correlational approach to investigate the relationships or patterns between the various identified cognitive structural elements. Empirical data regarding conceptual gaps will be collected from 23 student respondents using a specifically developed standardized test instrument.

Structurally, this framework aims to achieve two main outcomes: a comprehensive analysis of the causes of conceptual gaps and the formulation of strategic recommendations for learning interventions. The results of cognitive structure mapping and conceptual gap identification will serve as scientific evidence for formulating effective learning intervention strategies to address these gaps. Therefore, this study does not stop at diagnosing errors (conceptual gaps), but also includes an applicable solution phase through intervention recommendations. The quality of these recommendations directly depends on the accuracy of the collected data and the validity of the correlation analysis between students' cognitive structures and the manifestation of their conceptual gaps in fraction operations.

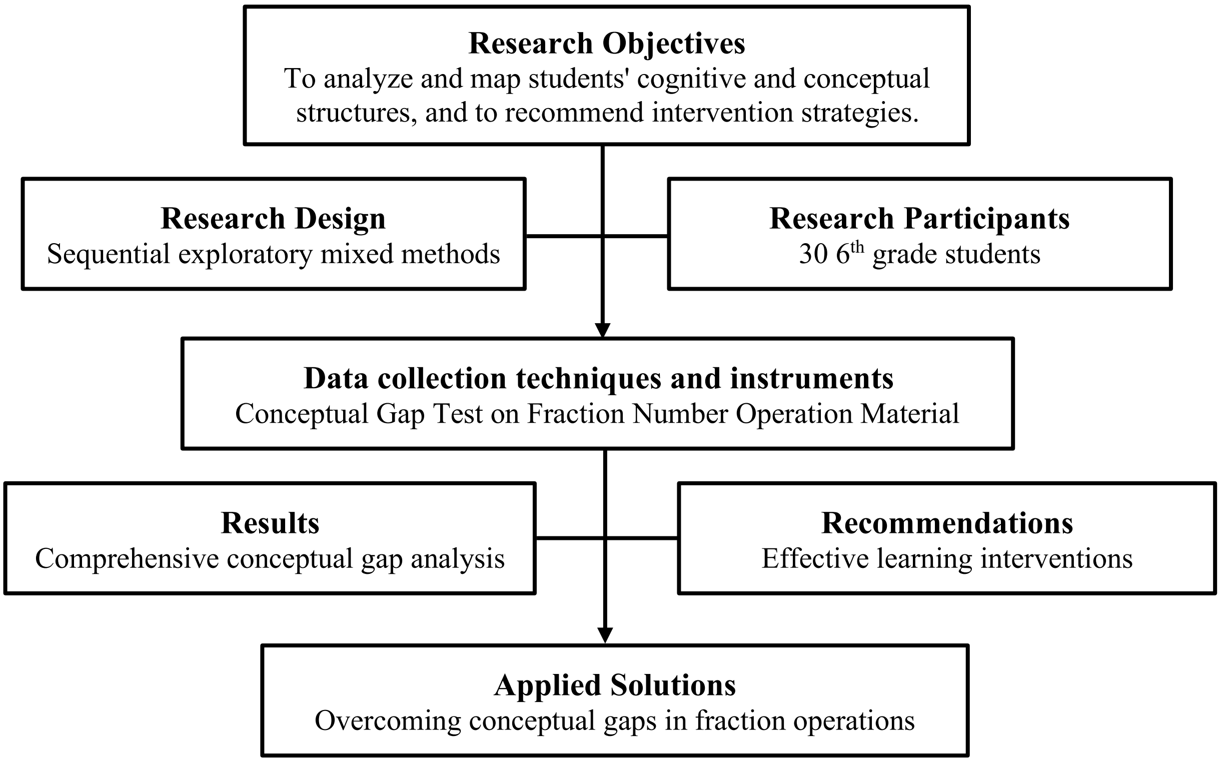

Figure 1. Conceptual Research Framework: Fractional Number Operations

METHOD

Research design

This study adopted a sequential exploratory mixed-methods approach to achieve its primary objective: to analyze and map students' cognitive structures in fraction operations that lead to conceptual gaps, and to recommend effective instructional intervention strategies to address these gaps. This methodological design, however, shifted focus from standard procedural understanding measurements to qualitative and diagnostic investigations. The initial phase of the study was qualitative-diagnostic, using specialized instruments and clinical interviews to map sixth-grade students' thinking patterns, primarily to identify underlying misconceptions and conceptual gaps. The results of this in-depth analysis served as the empirical foundation for designing the instructional intervention (Development Phase). Therefore, this study went beyond simple quantitative descriptions and sought to generate evidence-based solutions to bridge the diagnostically identified cognitive gaps (Plomp, 2013; Plomp & Nieveen, 2013).

Research location and participants

This research was conducted at an elementary school in Palembang, South Sumatra, Indonesia. Participants were 23 sixth-grade students (12 boys and 11 girls) selected using purposive sampling. The selection of sixth-grade students was based on the applicable national curriculum, as this level is the first formal introduction for students to the concept and operations of fractions. This criterion ensured that the subjects had a uniform learning experience relevant to the research focus. Demographically, the students were between 12 and 13 years old, which is a typical developmental age for students at this level.

Research instrument

The instrument used underwent content validity testing through expert judgment to ensure measurement accuracy for the High-Order Thinking Skills (HOTS) indicator. Specifically, the questions were adaptive modifications of HOTS items sourced from the PISA test and valid references tailored for elementary school students. The final instrument contained three items on fraction arithmetic operations, accompanied by clear and explicit instructions to guide respondents' understanding. The test content distribution was as follows: one item focused on measuring conceptual understanding in comparing fractions with unlike denominators; one item was designed to test procedural skills in arithmetic operations with fractions with unlike denominators; and the final item tested procedural skills in analyzing problems and determining the appropriate operation. To ensure content validity, these questions underwent an expert validation process involving critical input from experienced elementary school mathematics educators. These experts carefully evaluated the relevance, clarity, and alignment of each item with the research objectives. After the revision and validation process was completed, three final items were obtained.

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1. Three children, Ahmad, Budi, and Cici, are reading a book of the same thickness. Ahmad has read $3/4$ of the book, Budi has only read $2/5$ of the book, and Cici has read $1/2$ of the book. Which of them has read the most and which has read the least?

2. At the traditional market, Mother bought $1/2$ kg of granulated sugar. Because the sugar at the first stall looked whiter, Mother decided to buy another $3/4$ kg of granulated sugar at a different stall. If all the sugar was put into a storage container at home, what is the total weight of sugar (in kilograms) that Mother bought?

3. A pizza pan is cut into 8 equal slices. At a party, Mother gave 3 slices of pizza to Siti's friends. Because one of Siti's friends was too full, she returned one slice. Determine how many of the total pizzas were actually eaten at the party?

Data collection procedures

The implementation of this study required researchers to adhere to all established protocols, including securing official permission before data collection began, with test instruments distributed directly by the research team. To facilitate respondent understanding, the supervisor provided general instructions and specific directions before the session began, with respondents allotted between 75 and 90 minutes to complete all test items. Scoring was conducted using a four-level mathematics rubric; a three-point response reflects in-depth conceptual understanding, two points indicates partial understanding with minor errors, one point indicates limited understanding, and zero points are awarded for incorrect or irrelevant answers, including if the correct solution was obtained through an incorrect procedure. After all items were completed by respondents, the principal investigator was responsible for collecting all completed questionnaires.

Data analysis

The collected data were summarized and presented in tabular form. Frequencies, standard deviations (SD), and percentages were calculated. Arithmetic means were calculated to assess students' performance on the algebra test questionnaire involving fraction operations.

Ethical considerations

The researchers ensured that all respondents participated voluntarily, without coercion, after obtaining informed consent. Respondents were assured that the collected test data would be confidential and used solely for research purposes. To ensure ethical compliance, student and teacher consent was sought and obtained. In addition to respecting the right to privacy and anonymity, respondents' participation and contributions were duly acknowledged.

RESULTS

Based on the diagnostic test results, a striking competency gap was observed between indicators, with certain indicators showing lower levels of mastery than others. The student achievement profiles are summarized in Table 1.

Table 1. Frequency Distribution of Procedural Ability Levels in Fraction Arithmetic Operations

Level of Procedure Ability	Frequency	Percentage (%)
Poor	4	17,39
Fair	8	34,78
Good	10	43,47
Very Good	1	4,35
Total	23	100

Information:

Interval score	Description	
0 – 25	Poor	Number of cases (N) = 23;
26 – 50	Fair	Highest Score = 100;
51 – 75	Good	Lowest Score = 0;
76 – 100	Very Good	Mean = 47,52;
		Standard Deviation (SD) = 24,22.

Table 1. presents conceptual understanding in comparing fractions with different denominators among elementary school student respondents. The average conceptual understanding score for students was quite low, at 47.52. More than half (12 respondents or 52.17 percent) demonstrated low conceptual understanding. Only 11 respondents (47.82 percent) demonstrated an acceptable level of conceptual understanding.

DISCUSSION

Interpretation of the findings in this study provides a comprehensive overview of students' mastery of the material, particularly on fundamental topics. Data analysis shows that students' achievement has not yet reached the ideal standard, indicating a disconnect between mastery of calculation procedures and an understanding of fundamental mathematical logic. This presents a challenge for teachers in mathematics. Therefore, it is important for mathematics teachers to understand the causes of students' low conceptual understanding so they can begin taking small steps to address this issue.

Previous research indicates that students generally have a fairly good average level of conceptual understanding (Andamon & Tan, 2018), This contrasts with the results in this study, where students demonstrated "poor" conceptual understanding, as indicated by a mean conceptual understanding of 47.52 with a standard deviation of 24.22. Unlike previous research, this study focuses on fraction arithmetic operations, such as comparing fractions with different denominators, arithmetic operations on fractions with different denominators, and analyzing problems and determining appropriate operations.

The results of this study are supported by previous research which stated that students show a lack of conceptual and procedural understanding in dividing algebraic fractions (Cabuquin & Abocejo, 2024). Cabrillas (2010) in his research stated that students find the greatest difficulty in problems involving fractions. Furthermore, this poor conceptual understanding result is quite unfortunate considering that fraction arithmetic operations are an important foundation in mathematics that supports the mastery of more complex mathematical concepts at the next level, and has broad applications in solving everyday life problems (Kristesia et al., 2025). This needs to receive immediate attention by taking action to reduce this gap in understanding mathematical concepts, especially fraction arithmetic operations, so that students will now be more skilled in solving problems, especially in fraction arithmetic operations.

$\frac{1}{2},\frac{2}{5},\frac{3}{4}$ $\frac{1}{2},\frac{2}{3},\frac{4}{5}$ $\frac{1}{2},\frac{2}{5},\frac{3}{4}$ $\frac{1}{2},\frac{2}{5},\frac{3}{4}$

Figure 2. Example of Student Performance on the Conceptual Understanding Item: Comparing Fractions with Different Denominators

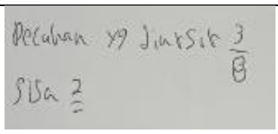
Figure 2. illustrates students' performance in fraction arithmetic operations in comparing fractions with different denominators. Several students showed a lack of understanding in comparing fractions with different denominators. The research findings indicate that there is a significant conceptual gap in grade 6 students, especially regarding fraction arithmetic operations involving different denominators. The dominant misconception found in several students is the tendency to compare fractions only based on the numerator or denominator partially (part comparison), not the relative value of the fraction as a whole. The diagnostic results indicate that students ignore the fundamental principle of fraction comparison, namely that comparisons are only valid if the fractions are in a uniform denominator condition (common denominator). Students assume $\frac{5}{2} > \frac{1}{2}$ because $2 > 1$ (comparing only the numerators) or $\frac{3}{4}$ is greater than the other because 4 is the largest denominator which may reflect a misconception about "dividing into more parts". This indicates that students' understanding of the concept of units and part-whole relationships in fractions is still weak. Students treat the numerator and denominator as two separate whole numbers rather than as a single ratio. This reinforces the notion that they approach problems as mechanical operations without contextual relevance, where procedures (such as equating denominators) are dismissed because their meaning is not understood.

$\frac{1}{2}+\frac{3}{4}=\frac{4}{6}$ $\frac{1}{2}+\frac{3}{4}=\frac{4}{6}$ $\frac{1}{2}+\frac{3}{4}=\frac{7}{3}$ $\frac{1}{2}+\frac{3}{4}=\frac{4}{6}$

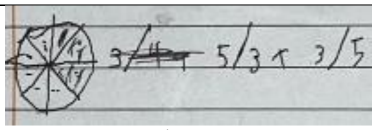
Figure 3. Example of Student Performance on Conceptual Understanding Question Items in Arithmetic Operations with Fractions with Unlike Denominators

Figure 3. presents elementary school students' procedural understanding of arithmetic operations with fractions with unlike denominators. Students appear to have a misconception about adding fractions by directly adding the numerators and denominators (e.g., $\frac{1}{2}+\frac{3}{4}=\frac{4}{6}$). The diagnostic results indicate several classic problems, starting with a lack of in-depth understanding of the

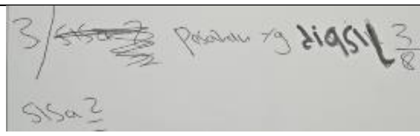
principles of fraction addition. Similar to Case 1 above, students fail to understand that the fractions to be added must have the same units. This likely stems from an overgeneralization of integer arithmetic operations, where they mistakenly apply the rules for adding integers to fractions, namely adding the numerators and denominators independently. The result of $4/6$ indicates that students only see the numbers involved: $1+3=4$ and $2+4=6$, without processing the conversion to a common denominator (LCM). This confirms the initial finding that students do not understand that adding fractions represents combining portions that must be of uniform size.



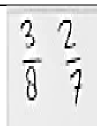
Shaded fraction $3/8$
Remainder 2.



$\frac{5}{3}+\frac{3}{5}$



Shaded fraction $3/8$
Remainder 2.



$\frac{3}{8};\frac{2}{7}$

Figure 4. Example of Student Performance on Conceptual Understanding Question Items: Analyzing Problems and Determining the Correct Operation

Figure 4. shows that students' conceptual gaps are not limited to procedural aspects but extend to the visual and contextual domains. In the context of questions involving visualization and change (subtracting parts), two crucial types of misconceptions were found: the error in determining the fraction from the image when the change occurs (case 3). Several students demonstrated the misconception that 3/8 becomes 2/7. In this case, when one part (the numerator) is reduced (from 3 to 2), the students incorrectly assume that the total number of parts (the denominator) also decreases from 8 to 7. This is a strong indicator that students fail to understand the fundamental definition of a denominator. They treat the denominator not as the total number of initial divisions of the whole unit, but as a variable that changes with the number of remaining parts. This misconception reflects a cognitive weakness in maintaining a constant denominator as a reference to the whole unit, resulting in students incorrectly subtracting $\frac{3}{8}-\frac{1}{8}$ to $\frac{3-1}{8-1}$.

Overall, these findings strongly corroborate previous findings (cases 1 and 2) regarding the failure to overgeneralize integer rules to fraction operations, which is a manifestation of students' lack of procedural understanding of fraction operations. This lack of procedural understanding is reflected in their tendency to compare fractions partially or add numerators and denominators independently, neglecting key procedures such as equating denominators. In addition to procedural problems, students also failed to grasp the principle of constant denominators when faced with dynamic visual contexts (case 3). Students' lack of conceptual understanding of operations in fraction operations, particularly in the operation of comparing fractions and adding fractions, is a factor that can cause difficulties in understanding other mathematical concepts, not only students' conceptual understanding of fractions.

Furthermore, they also struggle to correctly understand word problems, which results in errors in executing fraction arithmetic operations. The prevalence of these misconceptions can hinder their progress in mastering advanced mathematics topics and refining their problem-solving skills (Jankvist & Niss, 2018; Kusuma & Retnawati, 2019). Without understanding the basic principles of fraction arithmetic operations, students may face challenges in connecting algebraic concepts to real-world situations (Incikabi et al., 2020; Sujatha & Vinayakan, 2023). Addressing these misconceptions is crucial to provide a strong foundation for future mathematics learning and to foster deeper conceptual understanding.

Students' inability to establish connections between mathematical concepts can hinder their ability to apply prior knowledge to comparison problems involving fractions. This also suggests that students who cannot understand how different mathematical concepts relate to each other may struggle to transfer their knowledge and apply it to unfamiliar contexts. Strategies such as using illustrations from real-life contexts, explicitly emphasizing the logical connections between concepts

(Moreno et al., 2024), and providing opportunities for meaningful application of mathematical operations can play a crucial role in enhancing students' conceptual understanding.

This double gap, at both the procedural and visual-conceptual levels, suggests the need for intervention strategies that focus on reinforcing basic fraction concepts (especially the definitions of numerator and denominator) using concrete and visual models. Interventions should explicitly model that the denominator (the total fraction) only changes when the whole unit is re-divided, not simply because the number of parts of interest (the numerator) changes. Guided drills and practice can help improve their procedural fluency. Furthermore, students frequently demonstrate incorrect term cancellation in expressions and neglect parentheses. They mistakenly cancel unlike terms or terms with common factors, resulting in incorrect simplifications. The lack of accuracy in performing fraction operations suggests the need for students to pay more attention to detail during the problem-solving process. Mathematics teachers should emphasize grouping symbols and provide instruction to students on the appropriate timing for crossing out terms. This can contribute to improved procedural understanding and enhance students' problem-solving skills.

Given the fundamental role of procedural understanding in efficiently solving routine problems, educators can implement targeted instructional interventions to improve students' procedural fluency. Consistent and purposeful practice, accompanied by timely feedback, will strengthen their understanding and increase precision in mathematical operations. Educators can foster a deep appreciation for fraction operations by prioritizing procedural practice, ultimately strengthening students' problem-solving abilities. Equipping students with procedural fluency will empower them to approach more advanced mathematical topics with confidence and accuracy and prepare them for greater success in tackling complex challenges.

CONCLUSION

This study in-depth investigates the influence of conceptual understanding on fraction arithmetic operations among sixth-grade elementary school students. Key findings consistently indicate that respondents have a significant need for basic conceptual strengthening, as the quality of students' conceptual understanding is crucially correlated with the improvement of their overall mathematics abilities. Specifically, the stronger the conceptual understanding, the higher the students' demonstrated mathematical competence. The pedagogical implications of these results recommend that elementary school teachers proactively develop and enhance students' conceptual competence. This can be achieved through allocating adequate time for practice, reviewing problems, and providing meaningful exercises tailored to students' cognitive developmental stages. To optimize learning outside of class, periodically providing additional assignments is worth considering. In addition, mathematics teachers should routinely verify conceptual understanding by asking factual or conceptual questions and encouraging students to articulate their reasoning. Therefore, the teaching of mathematical concepts and procedures should be integrated simultaneously to maximize learning outcomes. However, limitations of this study must be acknowledged: the results obtained may be influenced by respondents' mathematics learning experiences prior to entering the target school. Given that this study was limited to a single elementary school, the findings may not be generalizable to all elementary schools in the country. Therefore, further research with a broader scope is needed to validate these findings. Future research could focus on identifying other external variables that influence elementary school students' conceptual understanding of mathematics.

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