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Patterns of Adaptive Mathematical Reasoning Errors Among Junior High School Students on The Pythagorean Theorem Based on Newman's Error Analysis

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Abstract: This study aims to analyze the adaptive mathematical reasoning abilities of eighth-grade junior high school students regarding the Pythagorean Theorem and to identify types of errors based on Newman's Error Analysis (NEA). A descriptive quantitative approach was used with 33 students selected through purposive sampling based on the recommendations of the mathematics teacher at the research site, as the students had studied the Pythagorean Theorem material which aligned with the research objectives, at a junior high school in Pekanbaru in October 2025. The instrument consisted of two open-ended questions containing four indicators: formulating conjectures, identifying patterns, providing mathematical justifications, and verifying the validity of arguments. Data were analyzed using mean scores, achievement percentages per indicator, and error classifications based on NEA. The results showed that the "formulating conjectures" indicator achieved the highest score (72.73%), while "identifying patterns" had the lowest (10.61%). The most dominant errors were encoding errors (97%) and comprehension errors (76%). It was concluded that students' adaptive mathematical reasoning skills remain relatively low, particularly in identifying patterns, providing mathematical justifications, and verifying the validity of arguments. These findings imply the need to emphasize learning activities that train pattern identification, mathematical justification, and argument verification, while also considering problem-based and inquiry-based instructional approaches as potential strategies to strengthen students' adaptive reasoning development.

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INTRODUCTION

Mathematics is one of the foundational disciplines that plays a crucial role in education, particularly in the 21st century, which demands higher-order thinking skills and flexibility in problem-solving (Sapri, 2023). This is consistent with the assertion that all fields of science, technology, engineering, and mathematics require mathematical proficiency that goes far beyond basic skills (Brewster & Miller, 2023), Nufus et al. (2024) also emphasize that education must actively respond to the challenges of globalization, rather than merely focusing on students' understanding of the trends and potential threats associated with it. The National Council of Teachers of Mathematics (NCTM) states that adaptive reasoning is one of the five main components of mathematical proficiency, along with conceptual understanding, strategic competence, procedural fluency, and productive dispositions (Kilpatrick et al., 2001). Adaptive reasoning holds a crucial position as it serves as the foundation for logical thinking, understanding relationships between concepts, considering various alternative solutions, and providing rational justifications for a conclusion (Kilpatrick et al., 2001). Adaptive reasoning acts as a determinant of the legitimacy of mathematical problem-solving strategies (Yanti et al., 2021). In the context of mathematics learning in schools, adaptive reasoning ability refers to four main indicators: the ability to make conjectures, identify patterns in mathematical phenomena, provide mathematical reasons or proofs, and verify the validity of an argument (Widjajanti, 2011). If students lack these abilities, they will tend to view mathematics as a difficult and meaningless subject (Setiani et al., 2024).

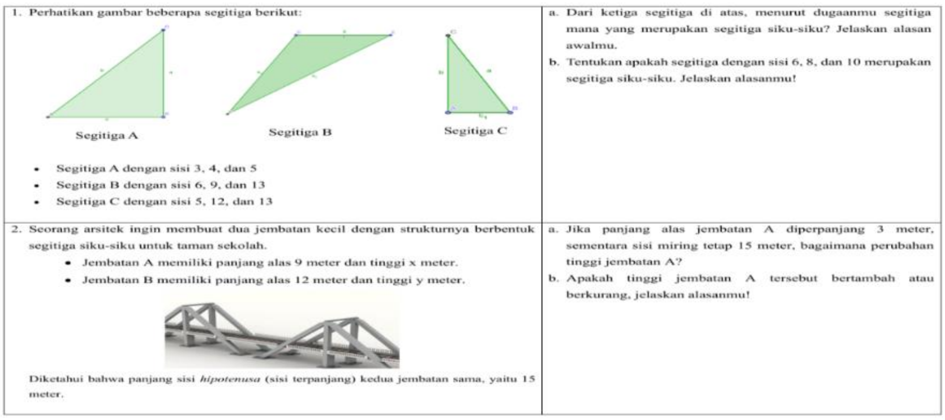
Students' adaptive mathematical reasoning skills in Indonesia remain relatively low. This aligns with the 2022 PISA results, which show that Indonesia's mathematics score was only 366 far below the OECD average of 472 with only 18% of students reaching Level 2 in mathematics (OECD, 2023). These results indirectly illustrate the concerning state of adaptive reasoning among Indonesian students. This situation reflects a broader challenge in mathematical literacy among Indonesian students. Habibi & Suparman (2020) emphasize that mathematical literacy in the context of 21st century skills demands reasoning ability, problem-solving, and decision-making competencies, areas in which Indonesian students have consistently shown low achievement in international assessments. This is reinforced by several studies, including those by Marbun et al. (2022), Wati (2025), Putri & Warmi (2024), which show that students' ability to engage in adaptive mathematical reasoning remains relatively low. Nurlita et al. (2025) in their systematic literature review, also state that most students struggle to solve adaptive reasoning problems due to limited prior knowledge, poor conceptual understanding, and a lack of familiarity with reasoning processes and justification in mathematics learning. This situation is also reflected in the researchers' preliminary findings from a survey of nine junior high school mathematics teachers in Riau. The results indicate that the indicator for formulating hypotheses is rated as good (scores 4–5), the indicators for providing mathematical reasons or evidence and drawing conclusions fall into the moderate category (scores 3–4), while the indicator for checking the validity of arguments is the lowest (scores 2–5). An interview with one of the teachers in Pekanbaru further reinforced this finding, indicating that students still have limitations in their basic mathematical abilities. This highlights the need for a more in-depth analysis of the nature and types of errors students make in adaptive reasoning.

An in-depth analysis of students' work is necessary to identify the sources and factors that hinder the development of adaptive reasoning. An appropriate approach for this purpose is Newman's Error Analysis (NEA). NEA is designed to comprehensively identify the sources of students' errors in solving math problems through five categories: reading errors, comprehension errors, transformation errors, process skills errors, and encoding errors (Rohmah & Sutiarso, 2018). This approach not only analyzes errors at the calculation stage, but also traces the process of understanding and representing mathematical problems from start to finish (Singh et al., 2010). Several previous studies have examined both students' adaptive mathematical reasoning and error analysis using the NEA, but separately. Studies on adaptive reasoning include those conducted by Ahsana Fitri et al. (2024), Wati (2025), and Putri & Warmi (2024) focuses on students' overall achievement without identifying the specific types of errors underlying their low achievement. Research by Ghefira &

Jupri (2025) indicates that NEA has proven effective in identifying errors made by junior high school students in geometry, but has not yet linked these to indicators of adaptive reasoning. Consequently, studies that specifically integrate these two approaches, adaptive reasoning analysis and NEA particularly regarding the Pythagorean Theorem at the junior high school level, remain very limited. This study is expected to provide a more comprehensive understanding of students' profiles of adaptive mathematical reasoning. Unlike previous studies that examined adaptive reasoning and Newman's Error Analysis (NEA) separately, this study integrates both approaches simultaneously to provide a more comprehensive diagnostic picture of students' errors in adaptive mathematical reasoning. To the best of the authors' knowledge, no prior study has specifically linked NEA error categories to adaptive reasoning indicators particularly formulating conjectures, identifying patterns, providing mathematical justifications, and verifying the validity of arguments in the context of the Pythagorean Theorem at the junior high school level in Pekanbaru. This integration constitutes the novelty of the present study. Based on the above, the research question in this study is: how do eighth-grade junior high school students' adaptive mathematical reasoning abilities in solving Pythagorean Theorem problems manifest when examined through the NEA approach? This study aims to analyze students' adaptive mathematical reasoning abilities based on indicators such as formulating conjectures, identifying patterns in mathematical phenomena, providing mathematical reasons or evidence, and verifying the validity of an argument, using Newman's Error Analysis (NEA) approach.

METHOD

This study employed a descriptive quantitative approach (Sugiyono, 2023). The research subjects were 33 eighth-grade students at a public junior high school in Pekanbaru, selected using purposive sampling based on the recommendations of the mathematics teacher. The purposive sampling criteria in this study were: (1) students had completed the Pythagorean Theorem topic, (2) the selected class was heterogeneous in terms of academic ability as confirmed by the mathematics teacher, and (3) students were present on the day of data collection. The sample was not selected based on academic achievement level, ensuring representation of students across varying ability levels." The study was conducted on October 21, 2025. The research instrument consisted of two open-ended questions on the Pythagorean Theorem designed to measure students' adaptive mathematical reasoning skills. Each question included more than one indicator of adaptive mathematical reasoning ability, namely: (1) proposing a hypothesis or conjecture, (2) identifying patterns in a mathematical phenomenon, (3) providing mathematical reasoning or proof, and (4) verifying the validity of an argument. The following are test questions on students' adaptive mathematical reasoning skills regarding the Pythagorean Theorem, intended for eighth-grade junior high school students in the first semester, presented in Figure 1.



1. Perhatikan gambar beberapa segitiga berikut: Segitiga A Segitiga B Segitiga C • Segitiga A dengan sisi 3, 4, dan 5 • Segitiga B dengan sisi 6, 9, dan 13 • Segitiga C dengan sisi 5, 12, dan 13	a. Dari ketiga segitiga di atas, menurut dugaanmu segitiga mana yang merupakan segitiga siku-siku? Jelaskan alasan awalmu. b. Tentukan apakah segitiga dengan sisi 6, 8, dan 10 merupakan segitiga siku-siku. Jelaskan alasanmu!
2. Seorang arsitek ingin membuat dua jembatan kecil dengan strukturnya berbentuk segitiga siku-siku untuk taman sekolah. • Jembatan A memiliki panjang alas 9 meter dan tinggi x meter. • Jembatan B memiliki panjang alas 12 meter dan tinggi y meter. Diketahui bahwa panjang sisi *hipotenusa* (sisi terpanjang) kedua jembatan sama, yaitu 15 meter.	a. Jika panjang alas jembatan A diperpanjang 3 meter, sementara sisi miring tetap 15 meter, bagaimana perubahan tinggi jembatan A? b. Apakah tinggi jembatan A tersebut bertambah atau berkurang, jelaskan alasanmu!

Figure 1. Adaptive Reasoning Test Item on Pythagorean Theorem

Table 1. Rubric for Assessing Mathematical Adaptive Reasoning Skills

Score	General Description
4	Able to reason, reflect, explain, and justify solutions completely and logically.
3	Able to reason and explain most steps correctly, but the argument is not yet fully sound.
2	Only presents calculations without justification or reflection.
1	Unable to relate the Pythagorean Theorem to the problem context or completely incorrect in calculations and reasoning.

Source: Adapted from Kilpatrick et al. (2001)

Item validation was conducted by soliciting expert opinions to assess the suitability and alignment of the items with the indicators being measured. The validation results indicate that all test items are suitable for measuring students' adaptive mathematical reasoning skills (Sugiyono, 2023). The primary research data were obtained from students' written work. Data analysis was conducted using descriptive quantitative methods by calculating the frequency, mean, and percentage of student achievement on each indicator of adaptive mathematical reasoning skills. Next, an analysis of student errors was conducted based on Newman's Error Analysis (NEA) framework by classifying students' answers into five types of errors: reading errors, comprehension errors, transformation errors, process skill errors, and encoding errors. This error classification was used to identify patterns of student errors in solving open-ended problems on the Pythagorean Theorem. The indicators for each type of error are presented in Table 2.

Table 2. Types of Errors and Indicators Based on NEA Procedures

Types of Errors	Indicator
Reading error	a. Students are unable to interpret symbols correctly b. Students are unable to identify the meaning of difficult words or terms in word problems (incorrectly copying information)
Comprehension error	a. Students do not write down what is given b. Students write down what is given but inaccurately c. Students do not write down what is asked d. Students write down what is asked but inaccurately
Transformation error	a. Students make a mistake in choosing the operation used to solve the problem b. Students make a mistake in modeling the problem into a mathematical form
Process skills error	a. Students incorrectly apply the appropriate mathematical rules or principles b. Students are unable to further process the solution to the problem c. Errors in performing calculations
Encoding error	a. Students make mistakes in writing the units of the final answer b. Students do not write the conclusion c. Students write the conclusion but it is incorrect

Source: Adapted from Suratih & Pujiastuti (2020).

RESULTS

The research results indicate that students' adaptive mathematical reasoning skills remain relatively low. The highest achievement was observed in the indicator of making conjectures (72.73%), while the indicators of identifying patterns (10.61%), providing mathematical reasons or evidence (14.39%), and verifying the validity of arguments (17.42%) fell into the very low category. This addresses the research problem and confirms that most students are not yet able to reason adaptively in a comprehensive manner, particularly at the higher-order reasoning stage, which requires relational understanding and mathematical justification. The average descriptive scores based on indicators of students' adaptive mathematical reasoning abilities at a junior high school in Pekanbaru are as follows on Table 3.

Table 3. Students' Adaptive Mathematical Reasoning Scores

Indicator	Score	Number of Students	Average Score
Formulating hypotheses or conjectures	4	17	2,91
	3	3	
	2	7	
	1	5	
	0	1	
Identifying patterns in mathematical phenomena	4	0	0,42
	3	1	
	2	0	
	1	11	
	0	21	
Providing mathematical reasoning or proof	4	2	0,58
	3	0	
	2	0	
	1	11	
	0	20	
Verifying the validity of an argument	4	2	0,70
	3	0	
	2	0	
	1	15	
	0	16	
Overall Average			1,15

Based on Table 3., it can be seen that the indicator with the lowest average score is "identifying patterns in mathematical phenomena," at 0.42. This score indicates that students' ability to recognize patterns and draw connections between concepts remains relatively low. Meanwhile, the indicator with the highest average score is "formulating hypotheses or conjectures," at 2.91. Furthermore, the percentage of students' performance in answering the given questions based on the indicators of adaptive mathematical reasoning ability can be seen in Figure 2.

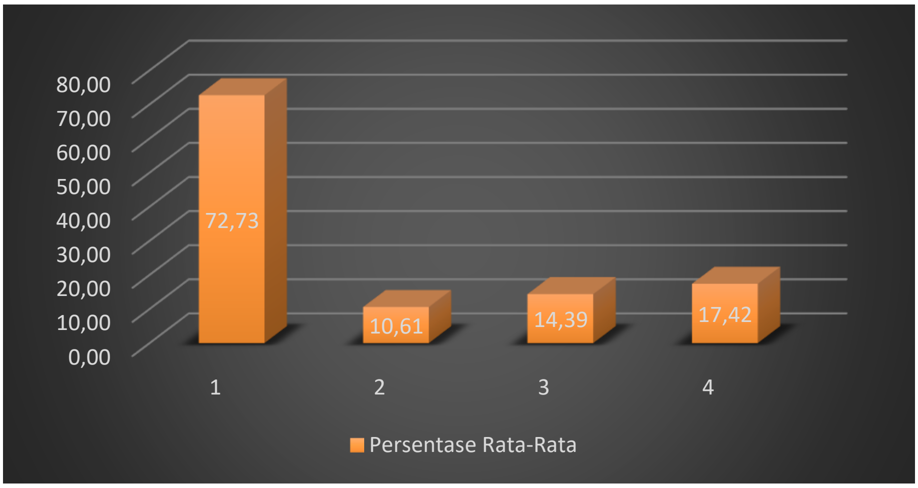


Figure 2. Percentage of Scores Based on Indicators of Adaptive Mathematical Reasoning Ability

Note:
1 : Formulating hypotheses or conjectures
2 : Identifying patterns in mathematical phenomena
3 : Providing mathematical reasoning or proof
4 : Verifying the validity of an argument

Figure 2. shows that the lowest percentage of student test scores was found in the indicator for identifying patterns in mathematical phenomena, at 10.61%; conversely, the test scores for the indicator of formulating hypotheses or conjectures had the highest percentage, at 72.73%. Next is the indicator for providing reasons or evidence at 14.39%, and the indicator for checking the validity of an argument reaches a percentage of 17.42%. Furthermore, the analysis of students' answers is presented in a more detailed overview of the types of errors students made at each stage of problem-solving, conducted through a further analysis of students' answers using Newman's Error Analysis (NEA). This analysis aims to identify the types of errors that occur during the process of solving math problems. The distribution of student error types based on Newman's Error Analysis is presented in Table 4.

Table 4. Percentage of Student Errors Based on Newman's Error Analysis (NEA)

Types of Errors	Number of Students	Error Rate
Reading	3	9.09%
Comprehension	25	75.76%
Transformation	10	30.30%
Process Skill	7	21.21%
Encoding	32	96.97%

This study applies Newman's Error Analysis (NEA) as a diagnostic procedure in which each student's This study applies NEA as a diagnostic procedure, in which all five error categories were assessed independently for each student's written response, without strictly following the hierarchical sequence proposed by Anne Newman. This means that even if a student made an error at an earlier stage, their response was still evaluated for errors at all subsequent stages. This approach is consistent with Kenney & Ntow (2024), who similarly applied NEA by assessing all five categories simultaneously within a single student response, rather than treating each stage as a sequential elimination gate. Consequently, the percentages of each error type in this study reflect the actual frequency of each error independently, which explains why encoding errors (96.97%) appear higher than comprehension errors (75.76%). Nearly all students failed to write a final conclusion, regardless of the errors they made at earlier stages. Table 4. shows that most student errors occurred during the question comprehension and final answer writing stages, while errors during the reading stage were relatively rare.

DISCUSSION

The results of a study conducted in eighth-grade junior high school classes in Pekanbaru on the Pythagorean Theorem are discussed below. The following is a discussion of the errors made by the study participants. Samuelsson (2010) in Awofala (2017) found that students' progress in conceptual understanding, strategic competence, and adaptive reasoning was significantly better when teachers used a problem-based curriculum compared to other instructional methods, suggesting that the low adaptive reasoning abilities observed in this study may be partly attributed to the conventional instructional approach experienced by the students.

Formulating hypotheses or conjectures

The indicator for formulating hypotheses or conjectures on open-ended question number 1 achieved the highest success rate, with 72.73% of students answering correctly. This indicates that most students were able to make logical initial estimates regarding which triangles were right triangles based on the given side lengths. Analysis of student responses shows that they were able to correctly identify Triangle A (3,4,5) and Triangle C (5,12,13) as right triangles, although some students did not fully explain the mathematical reasoning behind their answers. This aligns with the NEA category of "minor coding errors," where students are able to provide the correct answer but have not explicitly expressed their reasoning (or provided a justification based on the problem).

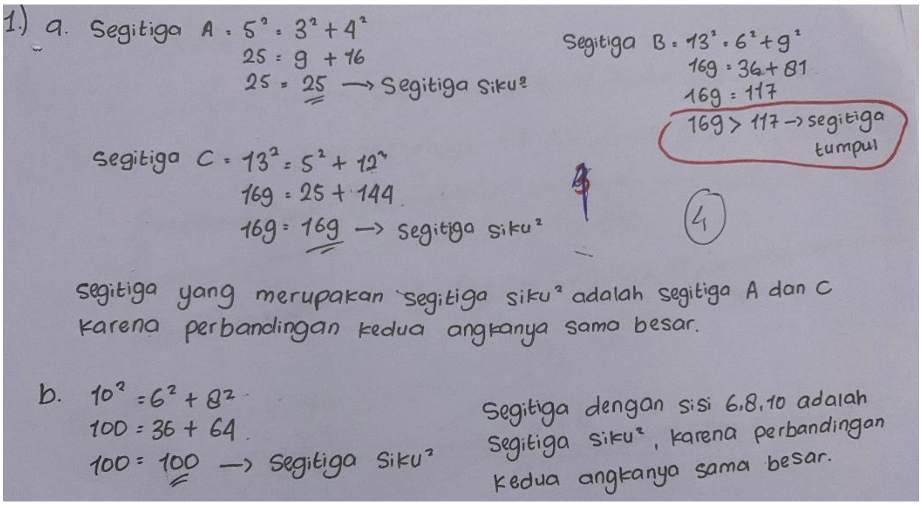


Figure 3. Answers from Students Who Answered Correctly and Completely

Figure 3. shows an example of a student's complete and correct answer. The test results indicate that 72.73% of students were able to answer correctly using the Pythagorean Theorem ($a^2 + b^2 = c^2$) and included a rationale or conclusion for their answer. In this answer, the student made no errors in reading, comprehension, transformation, processing skills, or final writing. Figure 3. represents similar answers from other students, although some students chose to guess the right-angled triangle based on visual observation or initial understanding, without a complete calculation. These findings are consistent with a study Darmayanti et al. (2022) of students with an accommodative learning style, in which both groups were able to propose initial hypotheses or strategies for solving problems. This aligns with Triyana & Gunawan (2025), who found that students' ability to complete all stages of problem-solving including evaluation and conclusion varied significantly based on their self-confidence level, with students showing strong initial understanding but inconsistent performance in writing final conclusions. Ansari et al. (2020) further support this finding, noting that while most students were able to propose initial conjectures, they struggled to find patterns and draw conclusions correctly, partly due to limited prerequisite knowledge and insufficient familiarity with higher-order reasoning processes.

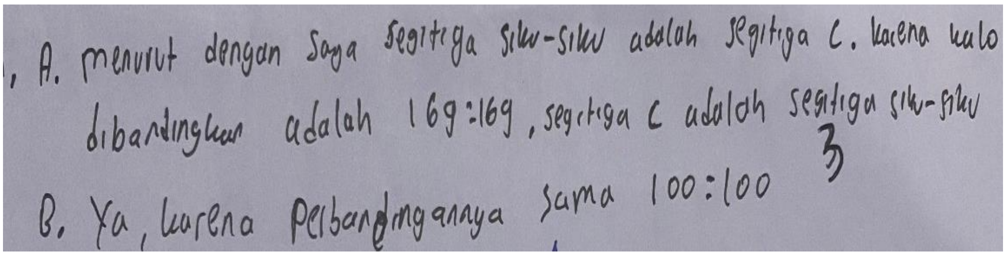


Figure 4. Students' Answers Based Solely on Guessing Without Mathematical Calculations

Based on Figure 4., the students were able to answer the questions correctly even though their answers were incomplete. In part a, question 1, the students only identified Triangle C as a right triangle, even though Triangle A also meets that criterion. However, the students' guess in part b was correct. The accuracy of this hypothesis indicates that the student has understood the concept of a right triangle based on the Pythagorean Theorem, namely that the square of the hypotenuse is equal to the sum of the squares of the two legs. This understanding is reflected in the student's statement: "Because when compared, it is $169 = 169$." When analyzed using Newman's Error Analysis (NEA), this falls under the categories of process skills errors and encoding errors. The student did not perform the calculation using the Pythagorean Theorem formula, so they failed to identify all the triangles that fit the criteria, resulting in an incomplete final answer. This is consistent with Kenney & Ntow (2024) who state that encoding errors occur when students are unable to provide accurate and complete final answersThe student's subsequent error on the same question also falls under the categories of process skills error and transformation error. Note Figure 5.

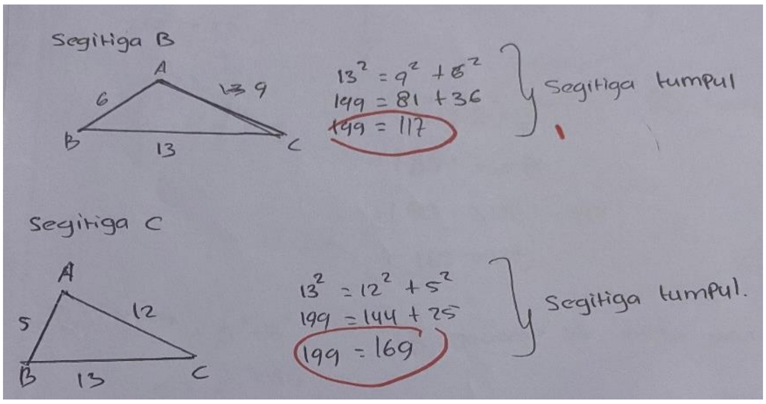


Figure 5. Student Errors in Calculations and Mathematical Modeling

The error shown in Figure 5. lies in the student's multiplication process when determining the value of 13^2. The student obtained the result 199, whereas the correct value is 169. This calculation error affected the student's hypothesis regarding the properties of the analyzed triangle, resulting in an inaccurate hypothesis. This finding indicates that the student's process skills are still weak. Additionally, students also made errors in the use of mathematical symbols. In Triangle B, the student wrote $199 = 117$, whereas the correct symbol should have been $199 > 117$. A similar error was also found in Triangle C. This indicates that students still face difficulties in modeling their hypotheses into the appropriate mathematical forms as shown in (Table 1.). These results align with previous research stating that students are capable of using relevant formulas, but carelessness in the calculation process can weaken the quality of the resulting answers (Ansari et al., 2020).

Identifying patterns in mathematical phenomena and providing mathematical justifications or proofs

The indicator of identifying patterns in mathematical phenomena is one of the adaptive reasoning skills that often shows low achievement, consistent with research (Haryadi & Oktaviana, 2021). In this study, question 2(a) combines the indicators of identifying patterns and providing mathematical reasoning or evidence. The results show that only about 10.61% of students were able to identify patterns, and 14.39% were able to provide mathematical reasoning or evidence, while the majority (about 90%) did not meet these indicators. This low percentage is not only due to students' inability to reason about relationships between variables but also because many students did not attempt the question at all, as evidenced by blank answer sheets. Nevertheless, the remaining students still attempted to answer, but none grasped the core of the question, which read: "If the base of Bridge A is extended by 3 meters, while the sloping side remains 15 meters, how does the height of Bridge A change?" This question encourages students to recognize the relationship between the sides of a triangle specifically the relationship between the base, height, and hypotenuse and to predict changes in height when the base changes. Note the students' answers in Figure 6.

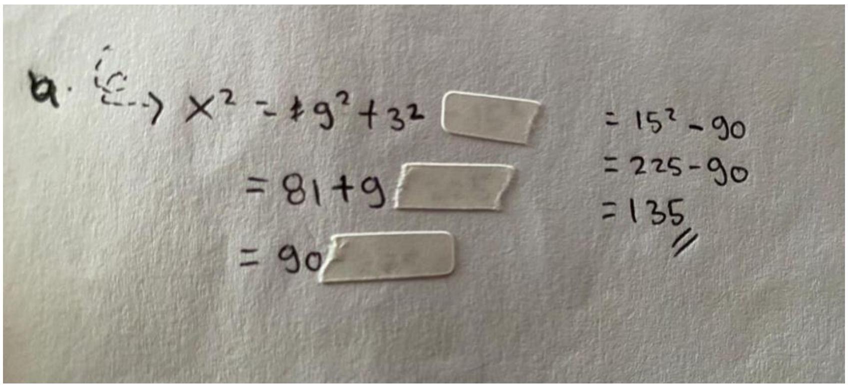


Figure 6. Student Responses to Question 2(a)

Based on the students' answers in Figure 6., it is evident that the students have not yet been able to identify the relationship between the base, height, and hypotenuse in a right triangle. The students wrote $x^2 = 9^2 + 3^2 = 90$, which indicates an inability to grasp the change in the base from 9 meters to 12 meters and the fact that the hypotenuse remains 15 meters. In terms of the indicator for identifying patterns, students cannot recognize that a change in one variable affects the other variables so that the Pythagorean relationship remains satisfied. In terms of the indicator for providing mathematical reasoning or proof, students wrote down calculations as mathematical proof but they were incorrect that is, they could not be justified because the students misunderstood the problem. The addition of 3 meters was treated as a new side in the Pythagorean theorem $(x^2 = 9^2 + 3^2)$, rather than as a change in the length of the base from 9 meters to 12 meters $(15^2 = 12^2 + b^2)$. As a result, the final answer does not address the question regarding how the bridge's height changes, and the final answer does not address the question regarding the change in the bridge's height. Based on Newman's Error Analysis, students experienced a comprehension error because they did not understand the intent of the question and the situation regarding the change in the base's length, a transformation error because they failed to model the problem into the correct mathematical form ($15^2 = 12^2 + b^2$), and an encoding error because the final answer did not align with the context of the question. Processing skills errors were not significant because the calculations were performed correctly, albeit within an incorrect model. This indicates that students have not met both indicators: identifying patterns and providing mathematical reasoning or evidence. This is consistent with the findings of Masitoh et al. (2024), who found that students in direct instruction demonstrated lower ability in identifying patterns and drawing conclusions from mathematical situations compared to students who learned using Problem-Based Learning. In addition to PBL, other problem-based learning models, such as Creative Problem Solving (CPS), have also been shown to improve students' adaptive reasoning skills (Faroh et al., 2022). These results indicate that problem-solving-centered learning can help students develop the ability to identify mathematical relationships, construct logical arguments, and draw accurate conclusions. In contrast to those findings, the students in this study still had difficulty identifying the relationship between the base, height, and sloping side of a triangle, so the mathematical reasoning they provided was not yet accurate.

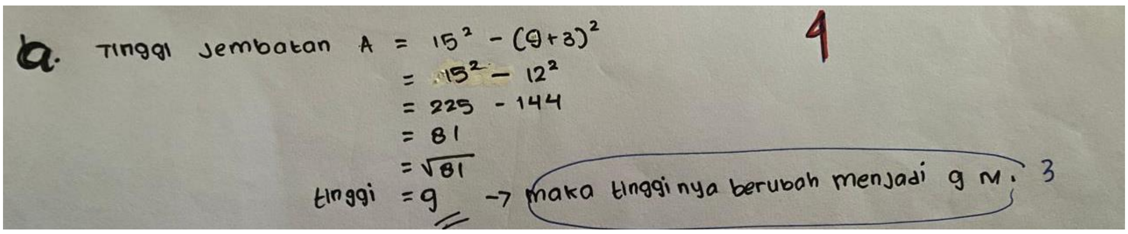


Figure 7. Student Responses According to the Indicators for Question 2a

Based on the students' answers in Figure 7., it is evident that the students used the Pythagorean Theorem to determine the height of the bridge after the base length changed from 9 meters to 12 meters, expressed as $15^2 - (9 + 3)^2$. Although the change in the base was not explicitly explained, substituting these new values indicates that the student understands the relationship between the base length and the bridge's height, as evidenced by the statement "the height changes to 9 m." This suggests the student has identified a pattern in the mathematical phenomenon, even if not yet symbolically or generally. The calculations and conclusions written also reflect an effort to provide mathematical reasoning and evidence, though they are not yet complete or systematic. The student's answer does not show reading errors, comprehension errors, transformation errors, or processing skills errors, as the problem was understood, the mathematical model is correct, and the calculations are accurate. However, there is an encoding error, namely the pattern is not written out completely, for example, "because the base changes, the height of the bridge also changes." This indicates that the student's ability to identify patterns still needs reinforcement, even though the indicator regarding providing mathematical reasoning or evidence has been met. These findings indicate that students' limited prior understanding affects their ability to model and identify relationships among elements within a problem. This is consistent with the results of a study conducted by Ahsana Fitri et al. (2024), which found that most students were unable to solve adaptive mathematical reasoning problems optimally due to limited prior knowledge and poor problem-comprehension skills. Dewi et al. (2020) also emphasize that students tend to be able to perform computational procedures but are not yet able to construct mathematical evidence or reasoning in a conceptual and precise manner within adaptive reasoning. This finding is consistent with Nur'Aisyah et al. (2021), who found that students' ability to provide mathematical reasoning or evidence and to verify the validity of arguments were the most difficult indicators to achieve, regardless of cognitive style suggesting that these higher-order reasoning skills require targeted instructional support. Lestari et al. (2022) also found that students who demonstrated strong initial competence were significantly more capable of constructing justifications and valid conclusions, reinforcing that justification is the most cognitively demanding indicator of adaptive reasoning and is strongly associated with students' prior mathematical knowledge.

Checking the validity of an argument

The indicator for checking the validity of an argument in question 2(b) achieved a percentage of 17.42%. This low achievement is due to the fact that most students did not answer question 2, leaving their answer sheets blank; in addition, many students were not precise in explicitly writing their conclusions and reasons, even though they had obtained the calculation results from the previous question. This finding is consistent with Mata-Pereira & Ponte (2017) which states that justifying and verifying the validity of arguments are the reasoning skills that are most difficult to facilitate in learning and are least often mastered by students on their own. Below is a student's answer that correctly applies the Pythagorean theorem, but the student is still unable to connect the result from question 2(a) to the prompt in question 2(b), which reads: "Does the height of Bridge A increase or decrease? Explain your reasoning!" This is because when answering question 2a, only about 10% of students were able to answer correctly, which affected their answers to question 2b. Note the students' answers in Figure 8.

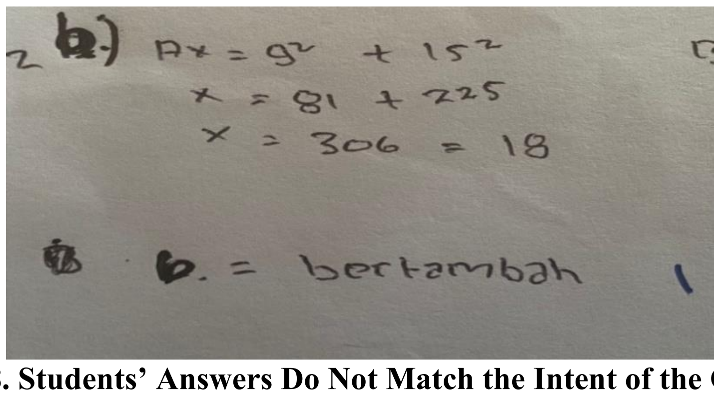


Figure 8. Students' Answers Do Not Match the Intent of the Question

The students' answers in Figure 8. reveal comprehension errors and processing skills errors according to Newman's Error Analysis (NEA). Comprehension errors arise because students do not know which parts to compare or the direction of change in the height of Bridge A after the base is extended. This led to a calculation error, where the student wrote $x = 9^2 + 15^2$ instead of using the correct equation $15^2 = 9^2 + a^2$. Consequently, the student failed to determine the height of the bridge correctly, compare the conditions before and after the base was extended, and verify the validity of the argument. These findings are consistent with research Darwani et al. (2020) which indicates that students still struggle to gather information from the problem and integrate it fully to assess the validity of the solution. This lack of understanding leads to errors in the calculation process. The student wrote $x = 9^2 + 15^2$, which demonstrates a mistake in applying the Pythagorean Theorem. In fact, based on the problem's information, the 15-meter length represents the hypotenuse, so the correct relationship should be expressed as $15^2 = 9^2 + a^2$. This error at this stage leads to an incorrect calculation of the bridge's height, preventing the student from comparing the conditions before and after the base is extended and failing to verify the validity of the argument regarding the change in the bridge's height. Unlike the previous student's answer, the student who answered correctly first determined the height of bridge A using the Pythagorean Theorem, namely $15^2 = 9^2 + a^2$, thereby obtaining the height of bridge A before the base was extended. Next, the student rechecked the height of the bridge after the base was extended to 12 meters, consistent with their answer to question 2a shown in Figure 7.

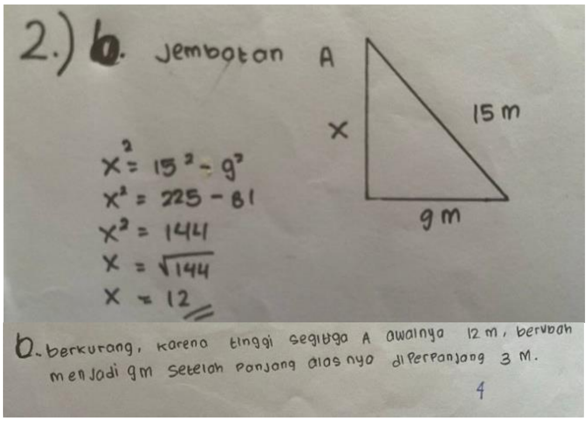


Figure 9. Student Responses That Meet the Validity of Argument Indicators

Based on Figure 9., the students' answers to question 2b indicate that they verified the validity of an argument by referring to the results of their previous calculations in question 2a. Students determined the initial height of the bridge to be 12 meters using the Pythagorean theorem $x^2 = 15^2 - 9^2$, resulting in an initial bridge height of 12 meters. They then checked the new height after the base length was extended to 12 meters in question 2a and obtained a height of 9 meters. The student concludes that the bridge's height decreased from 12 meters to 9 meters. This argument is valid because it is supported by consistent mathematical calculations and aligns with the Pythagorean relationship. Based on Newman's Error Analysis, the student did not experience reading errors, comprehension errors, or transformation errors because they were able to understand the problem and model it correctly, nor did they exhibit processing skills errors or encoding errors because the calculations and conclusions provided were consistent with the problem context. This indicates that the student has met the indicator of verifying the validity of an argument and providing appropriate mathematical reasoning and evidence. This is consistent with Farhan & Zulkarnain (2019), who found that students most frequently made errors at the transformation and process skill stages, while encoding errors also appeared consistently across different types of problems, reflecting students' difficulty in writing conclusions and final answers correctly. Jatisunda et al. (2024) similarly found that guided inquiry learning significantly improved students' adaptive reasoning ability, suggesting that reasoning skills particularly checking the validity of arguments require structured instructional scaffolding to develop.

CONCLUSION

The results of the study indicate that students' adaptive mathematical reasoning skills regarding the Pythagorean Theorem vary across each indicator. Students tend to be more capable of formulating hypotheses or conjectures, but struggle with identifying patterns, providing mathematical justifications or proofs, and verifying the validity of an argument. These difficulties are closely related to misunderstandings of the problem, mathematical modeling, and writing conclusions, as identified through Newman's Error Analysis. These findings suggest that learning the Pythagorean Theorem should place greater emphasis on activities that train students to identify patterns, provide mathematical justifications, and verify arguments through contextual problems, as well as on understanding the relationships between concepts and the ability to reflect on solutions mathematically. Teachers should also consider students' error types as a basis for determining appropriate learning strategies to improve their adaptive mathematical reasoning skills. Future research may further explore the integration of adaptive reasoning analysis with Newman's Error Analysis across different mathematical topics and broader student populations to strengthen the generalizability of these findings, and recommended to conduct experimental studies by implementing problem-based instructional approaches to empirically test whether these strategies can effectively improve students' adaptive reasoning abilities, particularly in the aspects of pattern identification, mathematical justification, and argument verification.

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